

JEE QUESTIONS - GRAVITATION

- 1) A satellite of mass 'm' orbits the Earth in a circular orbit of radius R. Its orbital speed is suddenly doubled without changing direction of motion

ANSWER

$$v = 2v_0 = 2\sqrt{\frac{GM}{R}}$$

$$E = \frac{v^2}{2} - \frac{GM}{R} = \frac{4GM}{2R} - \frac{GM}{R} = \frac{GM}{R}$$

$$L = m v_0 R = m(2v_0)R$$

$$= 2m v_0 R$$

For hyperbolic orbit:

$$r_p = \frac{L^2}{GMm^2} \frac{1}{1 + e} \quad \text{with} \quad e = \sqrt{1 + \frac{2EL^2}{GM^2m^3}}$$

Substitute $E = \frac{GMm}{R}$, $L = 2m v_0 R \rightarrow$ pericapsis $r_p = 3R$

$$\rightarrow \text{Apoapsis} \rightarrow \infty$$

$$E_{\text{new}} = \frac{GMm}{R}$$

- 2) Two planets A and B orbit the Sun in circular orbits with radii R & 4R

FIND

- Ratio of their orbital periods
- Ratio of their centripetal accelerations

ANSWER

Kepler's third law

$$T^2 \propto R^3$$

$$\frac{T_B^2}{T_A^2} = \frac{(4R)^3}{R^3} = 64 \Rightarrow \frac{T_B}{T_A} = 8$$

CENTRIPETAL ACCELERATION

$$a_c = \frac{v^2}{R} = \frac{(2\pi R/T)^2}{R} = \frac{4\pi^2 R}{T^2}$$

$$\frac{a_A}{a_B} = \frac{4\pi^2 R / T_A^2}{4\pi^2 (4R) / T_B^2} = \frac{1}{4} \times 64 = 16$$

$$\Rightarrow \frac{a_A}{a_B} = 16$$

3) A planet has mass M and radius R . Its surface gravity is g . A satellite is at height R above the surface.

FIND

- Escape velocity from height
- Compare v_e from the surface

ANSWER

$$v_e = \sqrt{2 \frac{GM}{R}}$$

$$\text{At surface, } v_0 = \sqrt{2gR}$$

$$\text{At height, } h=R, r=2R$$

$$v_n = \sqrt{\frac{2GM}{2R}} = \sqrt{\frac{GM}{R}} = \frac{v_0}{\sqrt{2}}$$

Q The ratio of escape velocity of a planet to the escape velocity of earth will be :-

Given: Mass of the planet is 16 times mass of earth and radius of the planet is 4 times the radius of earth

Sol: The escape velocity of a planet or a celestial body is given by:

$$v_e = \sqrt{\frac{2GM}{r}}$$

where $G \rightarrow$ gravitational constant, $M \rightarrow$ mass of the planet
 $r \rightarrow$ radius of the planet

Let subscripts 'p' and 'e' denote the planet and earth

$$\frac{v_{e,p}}{v_{e,e}} = \frac{\sqrt{\frac{2GM_p}{r_p}}}{\sqrt{\frac{2GM_e}{r_e}}} = \sqrt{\frac{M_p r_e}{M_e r_p}}$$

On substituting the given values

$$\frac{v_{e,p}}{v_{e,e}} = \sqrt{\frac{16 \cdot 1}{1 \cdot 4}} = \sqrt{4} = 2$$

$\therefore$ The ratio of the escape velocity of the planet to that of earth is 2:1

Q Two satellites A and B move round the earth in the same orbit. The mass of A is twice the mass of B. The quantity which is same for the two satellites will be

Sol: The quantity which is same for the two satellites will be speed.

The orbital speed of a satellite is independent of the mass of satellite, but it depends on the radius of the orbit. Potential energy, Kinetic energy and total energy depend on the mass of the satellite.

Q If V is the gravitational potential due to sphere of uniform density on its surface, then its value at the centre of sphere will be:-

Sol: The gravitational potential (V) due to a sphere of uniform density at a distance r from its centre is given by:

$$V(r) = \frac{GM}{2R^3} (3R^2 - r^2)$$

At the surface of the sphere ($r=R$), the gravitational potential is:

$$V = \frac{GM}{R}$$

Gravitational potential at the centre of sphere ($r=0$)

$$V(0) = \frac{GM}{2R^3} (3R^2 - 0) = \frac{3GM}{2R}$$

The gravitational potential at the centre of the sphere is $\frac{3}{2}$ times the potential at the surface of the sphere,

$$\therefore V(0) = \frac{3V}{2}$$

JEE Questions

①

Q.1) Consider a star of mass m_2 kg revolving in a circular orbit around another star of mass m_1 kg with $v_1 \gg v_2$. The heavier star slowly acquires mass from the lighter star at a constant rate of γ kg/s. In this transfer process, there is no loss of mass. If the separation between the centers of the stars is r , then its relative rate of change of $\frac{1}{r} \frac{dr}{dt}$ (ans^{-1}) is given by :- [JEE Advanced 2025]

(A) $\frac{-3\gamma}{2m_2}$

(B) $\frac{-2\gamma}{m_2}$

(C) $\frac{-2\gamma}{m_1}$

(D) $\frac{-3\gamma}{2m_1}$

Answer:- (*) Equilibrium is achieved through:-

$$m_2 \omega^2 r = \frac{G m_1 m_2}{r^2}$$

$$\omega = \sqrt{\frac{G m_1}{r^3}}$$

$$L = m_2 \omega^2 r^2 = m_2 \sqrt{\frac{G m_1}{r^3}} r^2$$

$$L = m_2 \sqrt{G m_1 r} = \text{constant}$$

$$L = \ln m_2 + \ln G + \frac{1}{2} \ln m_1 + \frac{1}{2} \ln r$$

$$L = \text{constant}$$

$$0 = \frac{dm_2}{m_2} + \frac{1}{2} \frac{dm_1}{m_1} + \frac{1}{2} \frac{dr}{r}$$

$$\Rightarrow \frac{1}{r} \frac{dr}{dt} = \frac{-2}{m_2} \frac{dm_2}{dt} = \frac{1}{m_1} \frac{dm_1}{dt}$$

(2)

$\Rightarrow$ 'y' transferred at a constant rate

$$\frac{dm_1}{dt} = y \text{ and } \frac{dm_2}{dt} = -y$$

$$\frac{1}{r} \frac{dr}{dt} = \frac{-2(-y)}{m_2} \frac{y}{m_1} \approx \frac{-2y}{m_2} s^{-1} \quad (B)$$

Q.2) Earth has mass 8 times and radius 2 times that of a planet. If the escape velocity from the earth is 11.2 km/s, the escape velocity in km/s from the planet will be:-

(A) 8.4

(B) 11.2

(C) 5.6

(D) 2.8

Ans: (C) $V_{e,e} = \sqrt{\frac{2GM}{R}} \Rightarrow 11.2$

$$V_{e,p} \Rightarrow \sqrt{\frac{2GM_p}{R_p}} = \sqrt{\frac{2G(M_e/8)}{R_e/2}}$$

$$V_{e,p} \Rightarrow \sqrt{\frac{GM_e}{2R_e}}$$

$$V_{e,p} = \frac{1}{\sqrt{2}} \times V_{e,e} \Rightarrow \frac{1}{\sqrt{2}} \times 11.2 \Rightarrow \underline{\underline{5.6 \text{ km/s}}} \quad (C)$$

Q.3) A satellite is moving with a constant speed 'v' in a circular orbit about the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection, the kinetic energy of the object is:- [JEE 2011 Advanced]

(A) $\frac{1}{2}mv^2$

(B) mv^2

(C) $\frac{3}{2}mv^2$

(D) $2mv^2$

Ans: (*) $T = K + U = 0$ { It escapes from gravitational pull of its total energy } (3)

$$K = -U \quad \text{--- (1)}$$

$$\frac{mv^2}{2} = \frac{GMm}{r}$$

$$r = \frac{GM}{v^2} \quad \text{--- (2)}$$

$$K = -U \Rightarrow -\left[-\frac{GMm}{r}\right] = \frac{GMm}{\frac{GM}{v^2}} = \underline{\underline{mv^2}} \quad \text{(B)}$$

Gravitation

1. A satellite of mass 200 kg is revolving in a circular orbit of radius $4R_E$ around the earth (R_E is the radius of the earth). Calculate the energy required to transfer it to an orbit of radius $8R_E$. Also, find the changes in its kinetic and potential energies ($R_E = 6400 \text{ km}$, $g = 9.8 \text{ m/s}^2$).

$\Rightarrow$ Total energy in a circular orbit is $E = -\frac{GMm}{2r}$.

$$\text{Initial Energy } (E_1) \text{ at } r_1 = 4R_E: E_1 = -\frac{GMm}{2(4R_E)} = -\frac{GMm}{8R_E}$$

$$\text{Final Energy } (E_2) \text{ at } r_2 = 8R_E: E_2 = -\frac{GMm}{2(8R_E)} = -\frac{GMm}{16R_E}$$

$$\Delta E = E_2 - E_1 = -\frac{GMm}{16R_E} - \left(-\frac{GMm}{8R_E} \right)$$

$$\therefore \Delta E = \frac{GMm}{16R_E}$$

Calculating Potential Energy Change (ΔU):

$$U = -\frac{GMm}{r}$$

$$\begin{aligned} \Delta U &= U_2 - U_1 \\ &= \left(-\frac{GMm}{8R_E} \right) - \left(-\frac{GMm}{4R_E} \right) \end{aligned}$$

$$= \frac{GMm}{8R_E}$$

Kinetic Energy Change (ΔK):

$$K = \frac{G M m}{2r}$$

$$\begin{aligned}\Delta K &= K_2 - K_1 \\ &= \frac{G M m}{2(8R_E)} - \frac{G M m}{2(4R_E)} \\ &= \frac{G M m}{16R_E} - \frac{G M m}{8R_E} \\ &= -\frac{G M m}{16R_E}\end{aligned}$$

Substituting Values,
 $G M = g R_E^2$ ($9.8 \times (6.4 \times 10^6)^2$) and $m = 200 \text{ kg}$

$$\underline{\Delta E \approx 1.57 \times 10^9 \text{ J}}$$

2. A satellite is revolving around the earth in a circular orbit of radius r . Derive an expression for the time period of the satellite and show that it is independent of the mass of the satellite.

$\Rightarrow$ The gravitational force provides the centripetal force $\frac{G M m}{r^2} = \frac{m r^2}{r}$

$$r = \sqrt{\frac{G M}{r}}$$

$$T = \frac{2\pi r}{v}$$

$$= 2\pi \sqrt{\frac{r^3}{G M}}$$

The expression for T is independent of m , hence the time period is independent of the mass of the satellite.

$$\therefore \text{The final answer is } 2\pi \sqrt{\frac{r^3}{GM}}$$

3. A body is projected vertically upwards from the surface of the earth with a velocity equal to half the escape velocity. Find the maximum height attained by the body.

$$\Rightarrow \frac{1}{2} m \left(\frac{v_e}{2} \right)^2 - \frac{GMm}{R} = -\frac{GMm}{R+h}$$

$$\frac{1}{2} m \left(\frac{1}{2} \sqrt{\frac{2GM}{R}} \right)^2 - \frac{GMm}{R} = -\frac{GMm}{R+h}$$

$$\frac{GMm}{4R} - \frac{GMm}{R} = -\frac{GMm}{R+h}$$

$$\frac{-3GMm}{4R} = -\frac{GMm}{R+h}$$

$$R+h = \frac{4R}{3}$$

$$h = \frac{R}{3}$$

Dhara K. Prajapati
XI-A
Roll no. - 12

GRAVITATION

Q1. Earth has mass 8 times & radius 2 times that of a planet. If the escape velocity from the Earth is 11.2 km/s, the escape velocity in km/s from the planet will be:-

Ans Planet: — $v_{ep} = \sqrt{\frac{2GM_p}{R_p}}$

Earth: — Mass $\rightarrow M_E$; Radius $\rightarrow R_E$; $v_{ee} = 11.2 \text{ km/s}$

$$\therefore v_{ee} = \sqrt{\frac{2GM_E}{R_E}} = 11.2 \text{ km/s}$$

Given, $M_p = \frac{M_E}{8}$; $R_p = \frac{R_E}{2}$

$$\therefore v_{ep} = \sqrt{\frac{2GM_p}{R_p}} = \sqrt{\frac{2G(M_E/8)}{R_E/2}} \Rightarrow \sqrt{\frac{2GM_E \cdot 2}{8R_E}} = \sqrt{\frac{GM_E}{2R_E}}$$

$$\therefore v_{ep} = \frac{1}{\sqrt{4}} \times v_{ee} = \underline{\underline{5.6 \text{ km/s}}}$$

Q.2 If a satellite orbiting the Earth is 9 times closer to the Earth than the Moon, what's the time period of rotation of the satellite? Given rotational time period of moon = 27 days & gravitational attraction b/w satellite & moon is neglected.

Ans. WKT, according to Kepler's Third Law, $\frac{T^2}{R^3} = \text{a constant.}$

Moon's distance from earth = R_{moon}

$T_{\text{moon}} = 27 \text{ days}$

$R_{\text{sat}} = (R_{\text{moon}}/9)$

acc. to Kepler's law: $\frac{T_{\text{sat}}^2}{R_{\text{sat}}^3} = \frac{T_{\text{moon}}^2}{R_{\text{moon}}^3} \Rightarrow \frac{T_{\text{sat}}^2}{\left(\frac{R_{\text{moon}}}{9}\right)^3} = \frac{27^2}{R_{\text{moon}}^3}$

$$\Rightarrow \frac{T_{\text{sat}}^2}{\frac{R_{\text{moon}}^3}{729}} = \frac{729}{R_{\text{moon}}^3} \Rightarrow \underline{\underline{T_{\text{sat}} = 1 \text{ day}}}$$

Q2 Match List - I & List II

- | | |
|-----------------------------------|-------------------------|
| A. Gravitational Constant | I. $[LT^{-2}]$ |
| B. Gravitational Potential Energy | II. $[L^2T^{-2}]$ |
| C. Gravitational Potential | III. $[ML^2T^{-2}]$ |
| D. Acc. due to gravity | IV. $[M^{-1}L^3T^{-2}]$ |

Ans $[G] = \frac{[F][r^2]}{[m_1][m_2]} = \frac{[MLT^{-2}][L]^2}{M^2} = \underline{[M^{-1}L^3T^{-2}]}$

$[U] = [mgh] = [MLT^{-2}L] = \underline{[ML^2T^{-2}]}$

$[\text{Gravitational Potential}] = [U]/[m] = \frac{[ML^2T^{-2}]}{[M]} = \underline{[L^2T^{-2}]}$

$[g] = \underline{[LT^{-2}]}$

∴ A-IV; B-II; C-II; D-I

Dhruv Rajpuri
XI A

gravitation

Q> A satellite launched into circular orbit of Radius 'R' around earth. A second satellite launched into an orbit of $1.03R$. The time period of revolution of second satellite is larger than first one approximately by:

- (A) 3% (B) 2.5% (C) 4.5% (D) 9%

Sol.

$$T = 2\pi \sqrt{\frac{R^3}{GM}} \rightarrow \text{Time period of revolution of body}$$

For 1st satellite:

$$T_1 = 2\pi \sqrt{\frac{R^3}{GM}}$$

For 2nd satellite:

$$T_2 = 2\pi \sqrt{\frac{(1.03R)^3}{GM}} \quad \{R' = 1.03R\}$$

$$T_2 = 2\pi \sqrt{\frac{R^3}{GM}} \times (1.03)^{3/2} \quad \left\{ T_1 = 2\pi \sqrt{\frac{R^3}{GM}} \right\}$$

$$T_2 = T_1 \times (1.03)^{3/2} = T_1 \times (1 + 0.03)^{3/2}$$

$$\text{Since in } (1+x)^n \text{ if } x \rightarrow (1+x)^n = 1 + nx$$

$$\text{So } T_2 = 1 + 0.03 \times \frac{3}{2} = 1 + 0.045$$

↳ 4.5%

∴ (C) 4.5%

Q) An object of mass 1 kg is taken to a height from surface of earth which is equal to 3 times radius of earth. The gain in potential energy of object will be: (if $g = 10 \text{ m/s}^2$ & radius of earth = 6400 km)

- (A) 48 MJ (B) 24 MJ (C) 36 MJ (D) 12 MJ

Sol.

$$\text{Initial PE} = U_i = -\frac{GMm}{R} \text{ \{at surface of Earth\}}$$

$$\text{final PE} = U_f = -\frac{GMm}{(4R)} \text{ \{at 3 times radius of Earth from surface\}}$$

$$\text{Gain in PE} = \text{final PE} - \text{initial PE}$$

$$\Delta U = U_f - U_i$$

$$\Delta U = -\frac{GMm}{4R} - \left(-\frac{GMm}{R}\right) = -\frac{GMm}{4R} + \frac{GMm \times 4}{R \times 4}$$

$$\Delta U = \frac{3GMm}{4R} = \frac{3}{4} mgR \left\{g = \frac{GM}{R^2}\right\}$$

$$\Delta U = \frac{3}{4} \times 1 \times 10 \times 64 \times 10^5 \text{ (6400 km} = 64 \times 10^5 \text{ m)}$$

$$\Delta U = 48 \times 10^3 \text{ J}$$

$$\Delta U = 3 \times 16 \times 10^6 = 48 \times 10^6 = \underline{\underline{48 \text{ MJ}}}$$

$$\boxed{\text{(A) 48 MJ}}$$

Q) The masses and radii of earth and moon are (m_1, R_1) & (m_2, R_2) respectively. Their centres are at distance 'r' apart. Find escape velocity for a particle of mass 'm' to be projected from middle of these 2 masses.

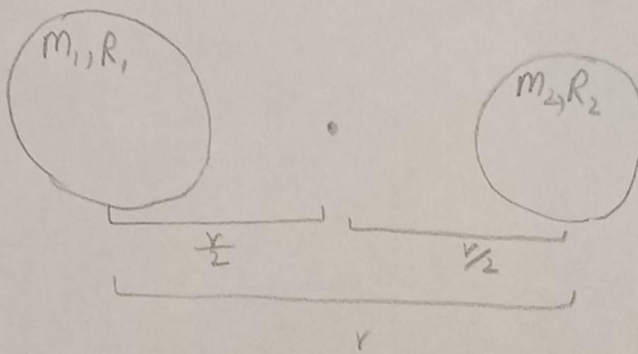
$$\text{(A) } V = \frac{1}{2} \sqrt{\frac{4G(m_1 + m_2)}{r}}$$

$$\text{(B) } V = \sqrt{\frac{4G(m_1 + m_2)}{r}}$$

$$\text{(C) } V = \frac{1}{2} \sqrt{\frac{2G(m_1 + m_2)}{r}}$$

$$\text{(D) } V = \sqrt{\frac{2G(m_1 + m_2)}{r}}$$

Sol:



For a particle of mass 'm' to escape ~~the~~ from middle of two masses:

$$\text{i.e. } KE + U_{\text{net}} = 0$$

$$\frac{1}{2}mv^2 + U_{\text{earth}} + U_{\text{moon}} = 0$$

$$\frac{1}{2}mv^2 - \frac{Gm_1m}{r/2} - \frac{Gm_2m}{r/2} = 0 \quad \left\{ R_1, R_2 \text{ won't matter as dist b/w them is } r \right\}$$

$$\frac{v^2}{2} = \frac{2Gm_1}{r} + \frac{2Gm_2}{r}$$

$$\frac{v^2}{2} = \frac{2G(m_1+m_2)}{r}$$

$$\frac{v^2}{2} = \frac{4G(m_1+m_2)}{r}$$

$$v = \sqrt{\frac{4G(m_1+m_2)}{r}}$$

$$\therefore V = \sqrt{\frac{4G(m_1+m_2)}{r}} \quad \textcircled{B}$$

Gravitation

Q1) An object is kept at rest at a distance of $3R$ above the Earth's surface where R is Earth's radius. The Minimum speed with which it must be projected so that it does not return to Earth is:

(Assume M = mass of Earth, G = Universal Gravitational Constant.)

Answer:-

$$V_0 = \sqrt{\frac{GM}{r}} \quad \bigg| \quad V_e = \sqrt{\frac{2GM}{r}}$$

$$r = 4R$$

$$\therefore V = -\frac{GMm}{r} \quad ; \quad K = \frac{1}{2}mv^2$$

$\therefore$ At ~~$4R$~~ $r = 4R$,

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r} = \frac{1}{2}mv^2 - \frac{GMm}{4R}$$

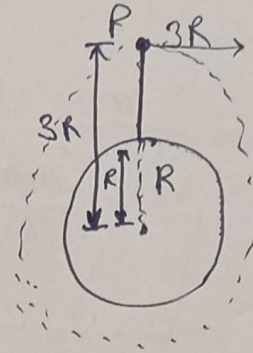
$\therefore$ At Infinity,

$$E = 0$$

$$\therefore \frac{1}{2}mv^2 - \frac{GMm}{4R} = 0$$

$$\Rightarrow v = \sqrt{\frac{GM}{2R}}$$

$$\boxed{\text{Answer}} \quad \therefore v_{\min} = \sqrt{\frac{GM}{2R}} \quad \left[\text{C} \sqrt{\frac{GM}{2R}} \right]$$



Q2) Earth has mass 8 times and radius 2 times that of a planet. If the escape velocity from Earth is 11.2 km/s , the escape velocity in km/s from the planet will be,

Answer:-

Let mass of Earth be m ; Mass of planet be M .
Let radius of Earth be r ; Radius of planet be R .

$$\therefore m = 8M; r = 2R.$$

$$\begin{aligned} \therefore V_e &= \sqrt{\frac{2mGr}{r}} = \sqrt{\frac{8M \times 2 \times G}{2R}} = \sqrt{\frac{8GM}{R}} = 2\sqrt{\frac{2GM}{R}} \\ &= 2 \times V_e = 2 \times 11.2 = 22.4 \text{ km/s} \end{aligned}$$

$$\therefore V_e = \sqrt{\frac{2mGr}{r}} = 11.2 \text{ km/s}.$$

$$\therefore V_e^p = \sqrt{\frac{2MR}{R}} = \sqrt{\frac{2 \times \frac{m}{8} \times G}{\frac{r}{2}}} = \sqrt{\frac{m \times G}{4 \times 2} \times \frac{2}{r}}$$

$$= \sqrt{\frac{mG}{2r}}.$$

$$\therefore V_e^p = x \times V_e.$$

$$\Rightarrow \frac{\sqrt{m \times G}}{\sqrt{2r}} = x \times \frac{\sqrt{2} \times \sqrt{m} \times \sqrt{G}}{\sqrt{8}}$$

$$\Rightarrow x = \frac{\sqrt{8} \times \sqrt{2}}{\sqrt{4} \times \sqrt{8}} \Rightarrow x = \frac{1}{2}.$$

$$\therefore V_e^p = \frac{1}{2} \times V_e = \frac{1}{2} \times 11.2 = 5.6 \text{ km/s}.$$

(Q3) A satellite is launched into a circular orbit of Radius 'R' around the Earth. A second satellite is launched into an orbit of radius $1.03R$. The time period of revolution of the second satellite is larger than the first one approx. by

Q3

Answer:-

$$r_1 = R; r_2 = 1.03R.$$

$$\therefore T_1 = \sqrt{\frac{r_1^3}{GM}} \times 2\pi = 2\pi \sqrt{\frac{R^3}{GM}}.$$

$$T_2 = \sqrt{\frac{r_2^3}{GM}} \times 2\pi = 2\pi \sqrt{\frac{(1.03)^3 R^3}{GM}}.$$

$$\therefore \frac{T_2}{T_1} = \frac{2\pi \times \sqrt{\frac{(1.03)^3 \times R^3}{GM}}}{2\pi \times \sqrt{\frac{R^3}{GM}}} = \frac{\sqrt{(1.03)^3} \times \cancel{\sqrt{R^3}} \times \cancel{\sqrt{GM}}}{\cancel{\sqrt{GM}} \times \cancel{\sqrt{R^3}}} = \sqrt{(1.03)^3}$$

$$\Rightarrow T_2 = 1.03 \times \sqrt{1.03} \times T_1 \Rightarrow \frac{T_2}{T_1} = \sqrt{(1.03)^3}.$$

$$\therefore (1.03)^{3/2} = (1 + 0.03)^{3/2} = 1 + \frac{3}{2} \times 0.03 = 1.045$$

$$\therefore T_2 = 1.045 T_1.$$

$$\therefore \text{Increase} = (1.045 - 1) \times 100 = \boxed{4.5\%}$$

GRAVITATION

(1) A satellite of mass m is launched vertically upward with an initial speed u from the surface of the earth. After it reaches height equal to Earth's radius R , it ejects a rocket of mass $\frac{m}{2}$ so that subsequently the satellite moves in a circular orbit. The KE energy of the rocket is (Earth's mass = M , gravitational constant = G)

- (A) $\frac{GMm}{20R}$ (B) $\frac{GMm}{10R}$ (C) $\frac{GMm}{5R}$ (D) $\frac{GMm}{2R}$

Ans:- Initial total energy = Kinetic + Potential

At height R , potential energy = $-\frac{GMm}{2R}$

Using conservation of energy and momentum, the rocket gains kinetic energy.

$$\text{Final KE} = \frac{GMm}{20R} \quad \text{option (A)}$$

(2) An object is kept at rest at a distance of $3R$ above Earth's surface (where R is Earth's radius). The escape velocity from that point is

- (A) $\sqrt{\frac{2GM}{R}}$ (B) $\sqrt{\frac{2GM}{4R}}$ (C) $\sqrt{\frac{2GM}{R}}$ (D) $\sqrt{\frac{2GM}{(R+3R)}}$

Ans:- Escape velocity formula $v_e = \sqrt{\frac{2GM}{r}}$

At height $3R$, distance from earth's center = $R + 3R = 4R$

$$v_e = \sqrt{\frac{2GM}{4R}} = \sqrt{\frac{GM}{2R}}$$

Answer:- option (B)

(3) The time period of a satellite orbiting, just above the Earth's surface

(A) $2\pi \sqrt{\frac{R}{g}}$

(B) $2\pi \sqrt{\frac{R^2}{g}}$

(C) $2\pi \sqrt{\frac{R}{GM}}$

(D) $2\pi \sqrt{\frac{R^3}{GM}}$

Answer: $T = 2\pi \sqrt{\frac{R^3}{GM}}$

for satellite just above Earth's surface, $r = R$

$$T = 2\pi \sqrt{\frac{R^3}{GM}}$$

using $g = \frac{GM}{R^2}$, substitute

$$T = 2\pi \sqrt{\frac{R^3}{gR^2}} = 2\pi \sqrt{\frac{R}{g}}$$

Option (A)

Gravitation

Q.1 A simple pendulum has a time period T_1 when on the earth's surface and T_2 when taken to a height R above the earth's surface, where R is the radius of the earth. The value of T_2 / T_1 is

Ans: The time period of a simple pendulum $= 2\pi \sqrt{\frac{l}{g}}$

On the surface of the earth $g_1 = Gm/R^2$

At a height R above the earth $g_2 = Gm/(2R)^2$

$$\left(\frac{g_1}{g_2}\right) = \frac{4}{1}$$

$$T = 2\pi \sqrt{\frac{l}{g}}$$

The time period of the surface of earth $T_1 = 2\pi \sqrt{\frac{l(R)^2}{Gm}}$

The time period of the surface of earth $T_2 = 2\pi \sqrt{\frac{l(2R)^2}{Gm}}$

$$T_2 / T_1 = 2$$

Q.2 Two bodies of masses m and $4m$ are placed at a distance r . The gravitational potential at a point on the line joining them where the gravitational field is zero is

Ans: Applying Newton's law of gravitation

$$\frac{(Gm \times 1)}{x^2} = \frac{(G \times 4m \times 1)}{(r-x)^2}$$

$$\frac{x^2}{(r-x)^2} = \frac{1}{4}$$

$$\frac{x}{(r-x)} = \frac{1}{2}$$

$$2x = r-x$$

$$\frac{r}{3} = x$$

$$V = \frac{-Gm}{\frac{r}{3}} + \frac{-G(4m)}{\left(\frac{r-x}{3}\right)} \\ = -9Gm/r$$

Q.3 The height at which the acceleration due to gravity becomes $\frac{g}{9}$, in terms of R , the radius of the earth is,

$$g' = g (R/R+h)^2$$

$$g' = \frac{g}{3}$$

$$\frac{g}{3} = g (R/R+h)^2$$

$$\frac{1}{3} = R/R+h$$

$$h = 2R$$

Gravitation.

Q1. four particles. each of mass m and are distant from each other move along a circle of radius ' R ' under the action of their mutual gravitation attraction. find speed.

Sol:

$$a = \sqrt{2} R$$

$$F_1 = \frac{GM^2}{R^2} = \frac{GM^2}{2R^2}$$

$$2F_1 \cos 45^\circ = \frac{GM^2}{\sqrt{2} R^2}$$

$$F_2 = \frac{GM^2}{(2R)^2} = \frac{GM^2}{4R^2} \quad \therefore F = \frac{GM^2}{R^2} \left(\frac{1}{\sqrt{2}} + \frac{1}{4} \right)$$

$$V^2 = \frac{GM}{R} \left(\frac{1}{\sqrt{2}} + \frac{1}{4} \right) = \frac{GM}{R} \cdot \frac{1+2\sqrt{2}}{4}$$

$$V = \sqrt{\frac{GM}{4R} (1+2\sqrt{2})}$$

Q2. The mass of a space ship is 1000 kg . Has to be launched into free space. $g = 10 \text{ m/s}^2$, $R = 6400 \text{ km}$.

Sol:

$$E = \frac{GMm}{R} = m g R \quad \begin{matrix} m = 1000 \text{ kg} \\ g = 10 \text{ m/s}^2 \end{matrix}$$

$$E = 1000 \times 10 \times 6.4 \times 10^6$$
$$E = 6.4 \times 10^{10} \text{ J}$$

Q3. What is the minimum energy required to launch a satellite of mass "m" from the surface of a planet of mass "M" and Radius "R" at an altitude of "2R"?

Sol:

$$E_1 = -\frac{GmM}{R}, \quad E_2 = \frac{-GmM}{2R} = -\frac{GmM}{2R}$$

$$\Delta E = E_2 - E_1 = \left(-\frac{GmM}{2R} \right) - \left(-\frac{GmM}{R} \right)$$

$$\boxed{\Delta E = \frac{GmM}{2R}}$$

- Q. Earth has a mass 8 times & a radius 2 times that of a planet. If the escape velocity from Earth is 11.2 km/s . What is the escape velocity from the planet.

[JEE Main 2022]

Sol: $V_e = \sqrt{\frac{2GM}{R}}$

$$\frac{V_p}{V_e} = \sqrt{\frac{M_p}{M_e} \times \frac{R_e}{R_p}}$$

$$M_e = 8M_p \quad R_e = 2R_p$$

$$\frac{V_p}{V_e} = \sqrt{\frac{1}{8} \times \frac{2}{1}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

$$V_p = \frac{11.2}{2} = \underline{\underline{5.6 \text{ km/s}}}$$

- Q. If a satellite orbiting the Earth is 9 times closer to the Earth than the Moon, what is the time period of rotation of the satellite? (Moon's period = 27 days)

Sol: Kepler's 3rd law: $T^2 \propto R^3$

[JEE Main 2024]

$$\left(\frac{T_s}{T_m}\right)^2 = \left(\frac{R_s}{R_m}\right)^3$$

$$\left(\frac{T_s}{27}\right)^2 = \left(\frac{1}{9}\right)^3 = \frac{1}{729}$$

$$\frac{T_s}{27} = \sqrt{\frac{1}{729}} = \frac{1}{27} \Rightarrow T_s = \underline{\underline{1 \text{ day}}}$$

- Q. A small mass m is at $2R$ from the center of a solid sphere of mass M & radius R . If a spherical part of radius $R/3$ is removed, find the ratio of original force F_1 to new force F_2 . [JEE Main 2021]

Sol: $F_1 = \frac{GMm}{(2R)^2} = \frac{GMm}{4R^2}$

$$\text{Removed Mass} = m' = M \times \left(\frac{R/3}{R}\right)^3 = \frac{M}{27}$$

distance of m from cavity center.

$$r' = 2R - \frac{2R}{3} = \frac{4R}{3}$$

Force of Gravity:

$$F_{\text{cav}} = \frac{G(M/27)m}{(4R/3)^2} = \frac{GMm}{27 \times \frac{16R^2}{9}} = \frac{GMm}{48R^2}$$

$$\text{Ratio: } \frac{F_1}{F_2} = \frac{1/4}{1/48} = \frac{12}{1} \quad \underline{\underline{\text{Ans: } 12:1}}$$

- Q) The escape velocity from spherical planet A is 10 km/s .
The escape velocity from another planet B whose density and radius are 10% of those of planet A is --- m/s
- (A) 1000 (B) $1000\sqrt{2}$
(C) $200\sqrt{5}$ (D) $100\sqrt{10}$

Ans - D

$$V_c = R \sqrt{\frac{8\pi G \rho}{3}} \propto R \sqrt{\rho}$$

Planet B: $\rho_B = 0.1 \rho_A$ $R_B = 0.1 R_A$

$$\frac{V_B}{V_A} = 0.1 \sqrt{0.1} = \frac{1}{\sqrt{10} \times 10}$$

$$V_B = 10 \times \frac{1}{10\sqrt{10}} = \boxed{100\sqrt{10} \text{ m/s}^{-1}}$$

- Q) Earth has mass 8 times and $r = 2$ times of that of a planet. If V_{escape} from Earth is 11.2 km/s the velocity in km/s from planet will be ---

$$\frac{V_{\text{escape planet}}}{V_{\text{escape Earth}}} = \sqrt{\frac{M_P}{M_E} \times \frac{R_E}{R_P}} = \frac{1}{2}$$

$$= \frac{1}{2} (11.2) = 5.6 \approx \boxed{6}$$

Q.) Two Satellites A and B go ~~round~~ round a planet in circular orbits having radii $4R$ and R respectively. If the speed A is $3v$, the speed B will be:

(A) $3v$

(B) $6v$

(C) $\frac{4}{3}v$

(D) $12v$

Ans: $v = \sqrt{\frac{GM}{R}}$

$$\frac{v_A}{v_B} = \frac{1}{2}$$

$$v_B = 2v_A = \boxed{6v}$$

Gravitation

3AANUI

Q A simple pendulum has a time period T_1 when on earth's surface and T_2 when taken to height R above the earth's surface, where R is radius of the earth. The value of T_2/T_1 is.

⇒ Time period of simple pendulum :

$$2\pi\sqrt{L/g}$$

$$g_1 = \frac{GM}{R^2} \quad ; \quad g_2 = \frac{GM}{(2R)^2} \quad \frac{g_1}{g_2} = \frac{4}{2}$$

$$T = 2\pi\sqrt{L/g}$$

$$T_1 = 2\pi\sqrt{\frac{L(R)^2}{GM}} \quad T_2 = 2\pi\sqrt{\frac{L(2R)^2}{GM}}$$

$$\boxed{\frac{T_2}{T_1} = 2}$$

Q A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the n^{th} power of R . If the period of revolution of particle is T .

⇒ Acc. to questⁿ central force is given by

$$F_c \propto 1/R^n \rightarrow F_c = k(1/R^n)$$

$$m\omega^2 R = k(1/R^n) \rightarrow m = \frac{(2\pi^2)}{T^2} = k\left(\frac{1}{R^{n+1}}\right)$$

OR

$$T^2 \propto R^{n+1}$$

$$\boxed{T \propto R^{(n+1)/2}} \rightarrow \underline{\underline{\text{Ans}}}$$

Q A satellite is moving in a circular orbit of radius r around the Earth. Suddenly, its speed is increased by the ratio of its new apogee distance to the original orbit radius.

⇒ For circular orbit.

$$v = \sqrt{\frac{GM}{r}}$$

$$v_1 = 1.25v$$

$$v^2 = (1.25)^2 \frac{GM}{r} = 1.5625 \frac{GM}{r}$$

$$E = \frac{1.5625}{2r} - \frac{GM}{r} = (0.78125 - 1) \frac{GM}{r}$$

$$E = 0.21875 \frac{GM}{r}$$

$$E = -\frac{GM}{2a}$$

$$\frac{GM}{2a} = 0.21875 \frac{GM}{r}$$

$$a = \frac{r}{0.4375}$$

$$ra = 2a - r$$

$$2(2.286r) - r$$

~~$$ra = 3.57r$$~~

$$| a = 2.286 |$$

~~$$ra = 3.57r$$~~

$$| a = 3.57 |$$

Q1) Suppose there is a planet that went around sun twice as fast as Earth. What would be its orbital size compared to Earth?

A) Time taken to revolve by earth, $T_E = 1 \text{ year}$

Orbital radius of earth = R_E

Time taken by planet for 1 revolution, $T_P = \frac{1}{2} \text{ year}$

Orbit radius of planet = R_P

Kepler's Law:

$$\left(\frac{R_P}{R_E}\right)^3 = \left(\frac{T_P}{T_E}\right)^2$$

$$\frac{R_P}{R_E} = \left(\frac{T_P}{T_E}\right)^{\frac{2}{3}}$$

$$= \left(\frac{1}{2}\right)^{\frac{2}{3}}$$

$$= \sqrt[3]{0.5^2}$$

$$= 0.63 //$$

0.63 times smaller than earth's radius.

Q2) Saturn is 29.5 times the earth year. How far is saturn from the sun if earth is $1.5 \times 10^8 \text{ km}$ away?

A) $R_E = 1.5 \times 10^{11} \text{ m}$, $T_S = 29.5 T_E$

Kepler's third Law,

$$T = \left(\frac{4\pi^2 r^3}{GM}\right)^{\frac{1}{2}}$$

$$\frac{r_S^3}{r_E^3} = \frac{T_S^2}{T_E^2}$$

$$r_S = r_E \left(\frac{T_S}{T_E}\right)^{\frac{2}{3}}$$

$$= \cancel{r_E} \cancel{(29.5)^{\frac{2}{3}}} 1.5 \times 10^{11} (29.5)^{\frac{2}{3}}$$

$$= 1.5 \times 9.55 \times 10^{11}$$

$$= 14.32 \times 10^{11} \text{ m,}$$

The distance is $14.32 \times 10^{11} \text{ m}$.

Q3) Two stars of one solar mass (2×10^{30} kg) are approaching each other for a head on collision. When they are a distance 10^9 km, their speeds are negligible. What is the speed with which they collide? radius of each star is 10^4 km.

A) Given, $M = 2 \times 10^{30}$ kg
 $R = 10^4$ km = 10^7 m
 $r = 10^9$ km = 10^{12} m
 $V = 0$

$$T.E = \frac{-GMM}{r} + \frac{1}{2} mV^2 = \frac{+GMM}{r}, \text{ when at } 10^9$$

About to collide:

$$r = 2R$$

$$\text{Total K.E} = \frac{1}{2} Mv^2 + \frac{1}{2} Mv^2 = Mv^2$$

$$\text{Total P.E.} = \frac{-GMM}{2R}$$

$$T.E = Mv^2 - \frac{GMM}{2R}$$

Law of conservation of Momentum

$$Mv^2 - \frac{GMM}{2R} = \frac{-GMM}{R}$$

$$v^2 - \frac{GM}{2R} = -\frac{GM}{R}$$

$$v^2 = GM \left(-\frac{1}{R} + \frac{1}{2R} \right)$$

$$= 6.67 \times 10^{-10} \times 2 \times 10^{30} \left[-\frac{1}{10^{12}} + \frac{1}{2 \times 10^7} \right]$$

$$= 13.34 \times 10^{19} (-10^{-12} + 5 \times 10^{-8})$$

$$\approx 6.67 \times 10^{12}$$

$$v = \sqrt{6.67 \times 10^{12}} \approx 2.58 \times 10^6 \text{ ms}^{-1} //$$

Final speed = 2.58×10^6 m/s.